Parameter Selection Techniques

Reading: Chapter 6

Motivation: Consider spring model

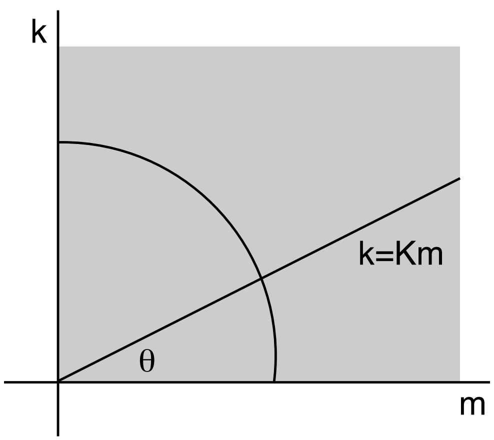
$$m\frac{d^2z}{dt^2} + kz = 0$$

$$z(0) = z_0 \;,\; \frac{dz}{dt}(0) = 0$$

with solution $z(t) = z_0 \cos(\sqrt{k/m} \cdot t)$.

Observation: Parameters q = [k,m] not uniquely determined by displacement data.

Admissible Parameter Space: $\mathbb{Q} = (0,\infty) \times (0,\infty)$



Note: Determination of slope equivalent to specifying θ

$$I(q) = \{\theta = \arctan(k/m) \,|\, 0 < \theta < \pi/2\}$$

$$NI(q) = \left\{r = \sqrt{k^2 + m^2} \,|\, r > 0\right\}$$

Note: $\mathbb{Q} = I(q) \oplus NI(q)$

- See Definitions 6.1 and 6.2 for definitions of identifiable and influential subspaces.

Linearly Parameterized Problems

Problem:

$$y = Aq \quad , \quad y \in \mathbb{R}^n \ , \ q \in \mathbb{R}^p$$

Nonlinearly Parameterized Problems

Problem:

$$y = f(q) \text{ or } y = f(t,q) \text{ or } y = f(x,q)$$

Note: Two basic strategies (global or pseudo-global sensitivity analysis)

- Variance-based (Sobol) methods
- Methods based on model linearization
  - Local sensitivity-based methods; e.g.,

    $$\chi_{ij}(q^*) = \frac{\partial f}{\partial q_j}(t_i, q^*)$$

  - Morris screening