MA 574 – PROJECT 2

Due: Wednesday, February 20

(1) Here we are going to use the MATLAB PDE toolbox to simulate acoustic scattering around a rectangular solid. As in the demo, let's consider a square with sides of 0.1 units and center at $[0.8, 0.5]$ that is rotated 45 degrees. The computational domain should be a circle with radius 0.45 units and the same center. We will consider two cases for the incident wave: (i) in the negative x direction with wavenumber $k = 60$ and (ii) from $\pi/4$ toward the center with the same wave number. For the wave equation in pressure, determine an appropriate boundary condition for the solid. You should use the Sommerfeld radiation condition on the circle.

Compute the solution for the two incident waves and, using either the online documentation or that posted on the class website, determine effective ways to plot your solution. You should make a movie of your results but you do not need to submit it.

(2) In this problem, we are going to develop the 1-D shallow water equations. These equations have been used to model waves in the atmosphere, rivers, lakes and oceans as well as gravity waves on a smaller domain (e.g., surface waves in a pool). One particular application is the modeling of tsunami.

Consider a channel of uniform width b and assume that the vertical velocity of the fluid is negligible (e.g., $w(t, x) = 0$) and horizontal velocity $u(t, x)$ is roughly constant through any vertical cross section. This is approximately true for small amplitude waves in a fluid that is shallow relative to its wavelength. We further assume that the fluid is incompressible so the density $\bar{\rho}$ is constant. Finally, we assume that the height $h(t, x)$ varies as a function of x as depicted in Figure 1.

(a) Quantify the mass and flux in the reference volume and use conservation of mass to determine a differential equation in terms of h and u.

(b) Now use conservation of momentum to derive a modeling equation that incorporates the effect of a pressure gradient in x. Check your units to ensure a proper balance of forces.

(c) The continuity and momentum equations contain the three unknowns u, h and p. To obtain an equation of state, we use a hydrostatic law which specifies that the pressure at height y is

$$p(y) = \bar{\rho} g y$$

where g is the gravitational constant. Integrate to specify the total force at (t, x) in terms of h. Reformulate the momentum equation solely in terms of the dependent variables h and u.

(d) Expand the derivatives in the momentum equation and use the continuity equation to simplify it. Formulate the continuity and momentum equations as a first-order system

$$\begin{bmatrix} h \\ u \end{bmatrix}_t + \begin{bmatrix} \varphi(h, u) \\ \psi(h, u) \end{bmatrix}_x = 0.$$

(e) Formulate appropriate boundary conditions along the sides of the stream.

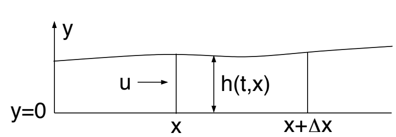

Figure 1: Geometry for the shallow water equations.