MA 574 – PROJECT 4

Due: Wednesay, April 17

(3) Consider a thin beam of length L with a fixed end at $x = 0$ as depicted in Figure 1. Let $\hat{\rho}, Y, C$ and γ respectively denote the density, Young's modulus, internal damping coefficient and air damping coefficient for the beam and let $w(t, x)$ denote transverse displacements. The beam decreases from thickness $2h$ at $x = 0$ to a thickness of h at the free end and has uniform width b. The z-axis is taken to be the vertical axis and the bottom of the beam has the coordinate $z = \frac{-h}{2}$. Finally, the beam has a mass m_L at $x = L$ along with a restraining spring with stiffness k_L that exerts a restoring force proportional to displacements $w(t, L)$.

(a) Define the neutral line and compute its position $n(x)$. Draw the neutral line on the figure of the beam.

(b) Determine relations for the linear density $\rho(x)$ and stiffness $YI(x)$.

(c) Balance moments and forces to obtain a strong formulation of the beam model.

(d) Specify appropriate boundary conditions.

(e) Integrate by parts to obtain a corresponding weak formulation. Be sure to specify the space of test functions.

(f) Consider the undamped beam ($\gamma = C = 0$) with $m_L = k_L = 0$. Use Hamiltonian (energy) principles to derive a weak formulation of the model. How does it compare with the model that you derived in (e)?

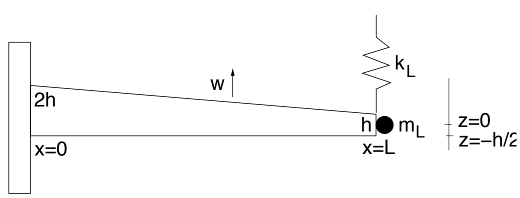

Figure 1: Thin vibrating beam.

(2) Consider the cantilever aluminum beam in the lab. Carefully measure its length, width, and thickness. Using the values $Y = 69$ GPa and $\rho = 2700$ kg/m^3 for aluminum, compute the first three natural frequencies in the absence of damping and compare to the experimental values.

(3) A string having mass per unit length ρ and length L is whirled about one end, with angular velocity ω, so that the motion is in a plane (neglect gravity). Using the property that the centripetal force exerted on a mass m moving in a circle of radius r with angular velocity ω is $F = mr\omega^2$, show that the tension in the string is

$$T(x) = \frac{1}{2}\rho\omega^2(L^2 - x^2),$$

where x is the distance from the stationary end. Show that the motion of the string is modeled by the differential equation

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(T\frac{\partial u}{\partial x}\right).$$

Specify appropriate boundary conditions.