Project 1 - Solutions

1.) Nonlinear equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x}$$

$$p = f(\rho)$$

Static Quantities

$\rho_0(t,x)$: Density not static

p_0 : Static pressure

$u_0 \neq 0$: Static velocity

$\Rightarrow p_0$ is constant in space

$$\Rightarrow \frac{\partial \rho}{\partial t} + u_0 \frac{\partial \rho}{\partial x} = 0$$

Perturbations:

$$\rho(t,x) = \rho_0(t,x) + \hat{\rho}(t,x)$$

$$p(t,x) = p_0 + \hat{p}(t,x)$$

$$u(t,x) = u_0 + \hat{u}(t,x)$$

Euler's Equation:

$$\rho_0 \frac{\partial \hat{u}}{\partial t} + \rho_0 u_0 \frac{\partial \hat{u}}{\partial x} = -\frac{\partial \hat{p}}{\partial x}$$

Continuity:

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \hat{\rho}}{\partial t} + u_0\left[\frac{\partial \hat{\rho}}{\partial x} + \frac{\partial \rho_0}{\partial x}\right] + \frac{\partial(\rho_0 \hat{u})}{\partial x} = 0$$

State: $\hat{p} = c^2 \hat{\rho}$

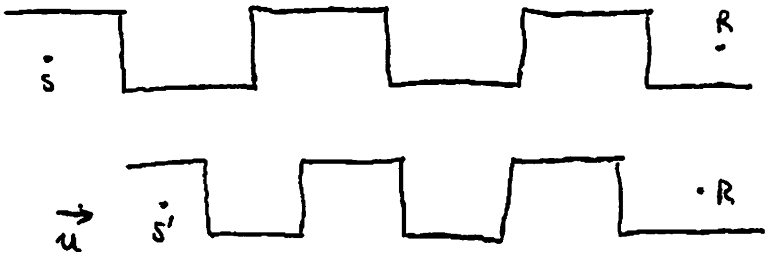
Note: No apparent way to formulate in terms of one state.

2.) The noise signal is

$$\text{Noise} = 1{,}85\sin(304\cdot 2\pi\cdot t + \pi/7) + 0.8\sin\left(98\cdot 2\pi t + \frac{\pi}{7}\right).$$

The piece is Handel's Hallelujah Chorus.

3.)



Note: $SR - S'R = u\Delta t$

$\Rightarrow \lambda_e f_e \Delta t - \lambda_o f_e \Delta t = u\Delta t$

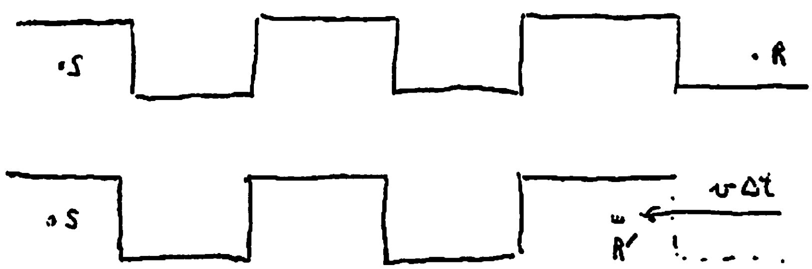
Note: $\frac{m}{wave} \cdot \frac{wave}{s} \cdot s = m$

$\Rightarrow \lambda_o = \frac{f_e \lambda_e - u}{f_e}$

However, $c = f_e \lambda_e = f_o \lambda_o \Rightarrow f_o = \frac{f_e \lambda_e}{\lambda_o} \Rightarrow \underline{\underline{f_o = \frac{c}{c-u} f_e}}$

Note: This increases the frequence as the source approaches and decreases it as they separate.

b) Reverse the role of the source and observer.



Here $SR - SR' = v\Delta t$

$\Rightarrow \lambda_e f_e \Delta t - \lambda_e f_o \Delta t = v\Delta t$

$\Rightarrow \underline{\underline{f_o = \frac{c-v}{c} f_e}}$

Note: The moving observer experiences more waves than the stationary one so f_o changes.

c.) Using $f_o = \frac{c}{c-u} f_e$ we get $293.665 = \frac{328.25}{328.25-u} f_e$

$233.082 = \frac{328.25}{328+u} f_e$

$\Rightarrow u = 37.75$ m/s

$\simeq 84.5$ mph

when we use the $-5°C$ value $c = 328.25$ m/s.