MA 573 — PROJECT 6

Due: Monday, November 20

(1) Consider the Sinko–Streifer model

$$\frac{\partial u}{\partial t} + \frac{\partial (gu)}{\partial x} = -\mu u$$
$$g(t, x_0)u(t, x_0) = R(t)$$
$$u(0, x) = \Phi(x)$$

with the growth function

$$\frac{dx}{dt} = g(x) = \frac{x_1 - x}{x_1 - x_0}$$

where x_0 and x_1 are the smallest and largest fish sizes. Determine the solution $u(t, x)$ for the initial condition regime $t \leq G(x)$ with the assumption that $\mu = \bar{\mu}$ is constant. You can take $x_0 = 1$ cm, $x_1 = 10$ cm and

$$\Phi(x) = \sin\left(\frac{\pi(x - x_0)}{x_1 - x_0}\right).$$

Plot the curve $G(x)$ along with a representative trajectory for $u(t, x)$.

(2) Consider the SIR epidemic model

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \nu I$$
$$\frac{dR}{dt} = \nu I$$

where β and ν denote contact and recovery rates. You can use the values $\beta = 4$ and $\nu = 1.2$.

(a) Simulate the spread of the epidemic using the initial conditions $S(0) = .99, I(0) = 0.01$ and $R(0) = 0$. When does it end?

(b) Now modify your model to include 90% vaccination of all individuals that provides life-time immunity. Simulate your results using the same initial conditions. Does your vaccination program work?