MA 573 — PROJECT 1

Due: Friday, August 25

The goal of this project is to acquaint you with Matlab and LaTeX. In class we discussed the linear spring model

$$m\frac{d^2y}{dt^2}(t) + c\frac{dy}{dt}(t) + ky(t) = f(t)$$
$$y(0) = y_0 \ , \ \frac{dy}{dt}(0) = y_1, \tag{1}$$

where m, c and k respectively denote the mass, damping and stiffness coefficients. For $f(t) = 0$, we know that the analytic solution to (1) is

$$y(t) = e^{-ct/2m}[a_1 \cos(\nu t) + a_2 \sin(\nu t)], \tag{2}$$

where

$$\nu = \frac{\sqrt{4km - c^2}}{2m} \ , \ a_1 = y_0 \ , \ a_2 = \left(y_1 + \frac{c}{2m}y_0\right)/\nu.$$

To numerically approximate the solution to (1), we discussed the implicit Euler algorithm

$$\vec{z}_{j+1} = [I - \Delta t A]^{-1}\vec{z}_j + [I - \Delta t A]^{-1}\Delta t \vec{F}(t_{j+1}) \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \quad , \quad \vec{F}(t) = \begin{bmatrix} 0 \\ f(t)/m \end{bmatrix}$$

and $\vec{z} = [y \, , \, \dot{y}]^T$. The true and approximate solutions obtained with $m = 4, c = 2, k = 16, f(t) = 0$ and $y_0 = 2, y_1 = 30$ are plotted in Figure 1.

Assignment: Implement the trapezoid method and `ode23` for the model (1) and write up your results in LaTeX. Compare the convergence rate of the trapezoid method with that of the implicit Euler algorithm and include a plot of your results. Include a table of results illustrating that you get the correct convergence rate when you double the number of gridpoints. You can work together on this in groups of up to four but submit individual reports.

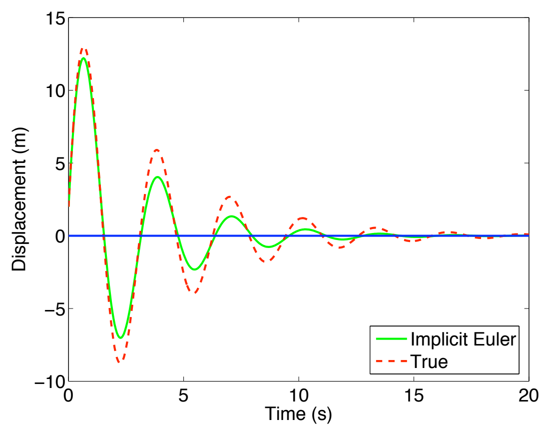

Figure 1: True and approximate solutions to the spring model (1).