MA 573 — PROJECT 4

Due: Monday, October 16

Problem 1.

In class, we showed that the solution to the diffusion model

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \rho}{\partial r}\right) \\ \rho(0,r) &= M\delta_0(r) \end{aligned} \tag{1}$$

is

$$\rho(t,r) = \frac{M}{\sqrt{(4\pi D t)^3}}e^{-r^2/4Dt}. \tag{2}$$

We also collected the data in Table 1 where times are reported in seconds and distances in both inches and meters. You should use the metric measurements for your computations.

For this project, you can assume that $\rho(t,r) = 1$ when the durian smell was first noted. For each (t,r) pair, (2) then has the form

$$1 - M \cdot f(D) = 0,$$

where $f(D)$ is the Gaussian function evaluated for specific values of t and r.

(i) We will initially solve for D and take $M = 1000$. The easiest way to do this is to plot the equation for a range of D-values and zoom in on the region where it crosses the axis. This zooming process can be done with arbitrary accuracy so you can get a good estimate for D. Note that there will likely be two roots so you can choose the first. Report the five values that you recover for D and discuss assumptions and sources of error.

(ii) Try varying the value of M and discuss its effect on your solution. You do not need to do this here, but you would typically estimate it in addition to D.

(iii) The diffusion constant for certain gasses is reported to be 2×10^{-5} m^2/s. How do your results compare to this value?

t (s)	r (inches)	r (m)
2.1	62	
27.1	88	
41.4	115	
96.6	153	
112	170	

Table 1: Times and distances noted for the durian scent.