Project 3 Solutions

1. For the Lincoln tunnel, we obtain $u_{max} = 15.895$ m/s $\simeq 35.3$ mph and $M = 0.1134$ cars/m. The differential equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left[\rho u_{max}\left(1 - \frac{\rho}{M}\right)\right] = 0.$$

2a) The rate is

$$\int_0^{2\pi}\int_0^{15} 0.1\, d\Omega = \pi(15)^2(0.1) = 22.5\pi \simeq 70.68 \frac{kg}{s}.$$

b) If ρ is constant, mass conservation $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$ is equivalent to $\nabla\cdot\vec{u} = 0$, which is satisfied here. To incorporate rip currents, you add an x_1-component to the velocity.

c) The model is

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\vec{q} = b \quad \text{where } \vec{q} = \rho\vec{u} - D\nabla\rho$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \rho\nabla\cdot\vec{u} + \vec{u}\cdot\nabla\rho = D\Delta\rho + b$$

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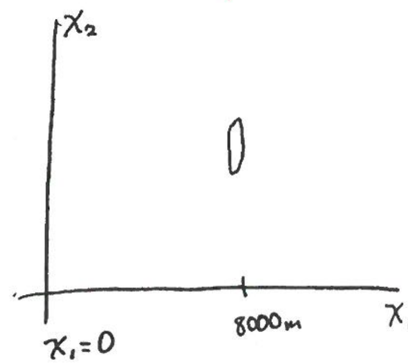

BC: $x_1 = 0 \quad \vec{q}\cdot\langle -1, 0\rangle = 0$

$$\Rightarrow -\rho u_1 + D\frac{\partial \rho}{\partial x_1} = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial x_1}(0, x_2, t) = 0$$

Also, $\rho(\infty, x_2, t) = \rho(x_1, \pm\infty, t) = 0$

IC: $\rho(x_1, x_2, 0) = 0$ if water is unpolluted

3, 4, 5: Numerical