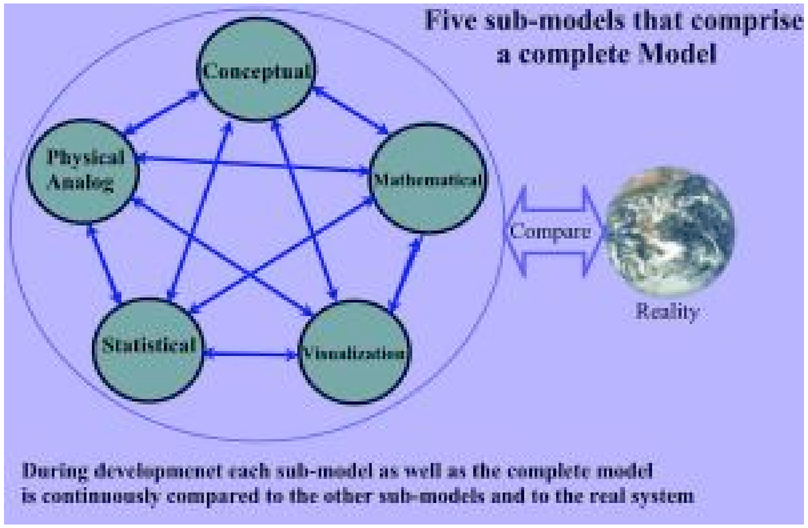
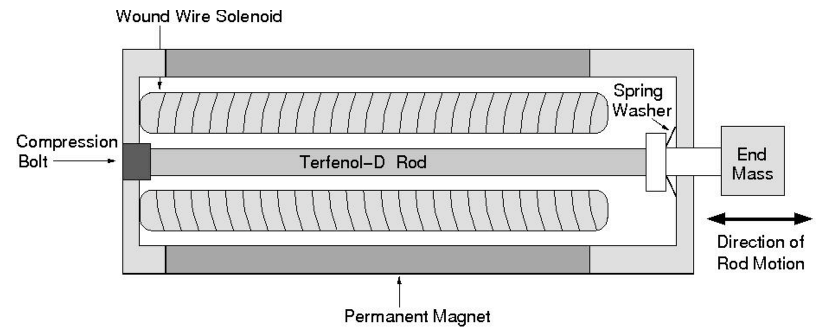
Introduction and Motivation

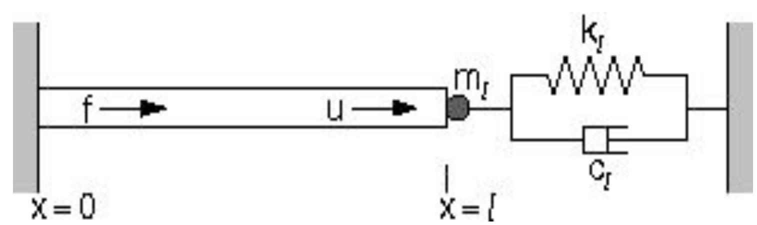
``Essentially all models are wrong but some are useful,''
George E.P. Box, Industrial Statistician



Terfenol-D Transducer

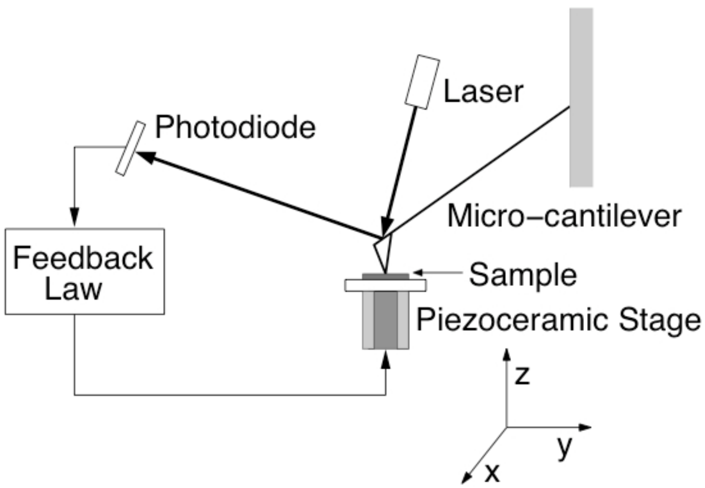
- Schematic of Terfenol-D transducer in the SolidDrive
- Terfenol-D rod can be modeled as a rod with elastic and damped boundary conditions.
- For uniform input fields, spring equation with nonlinear inputs provides “good” approximation of rod dynamics.
- What is nonlinear and hysteretic behavior of f(H,u)?

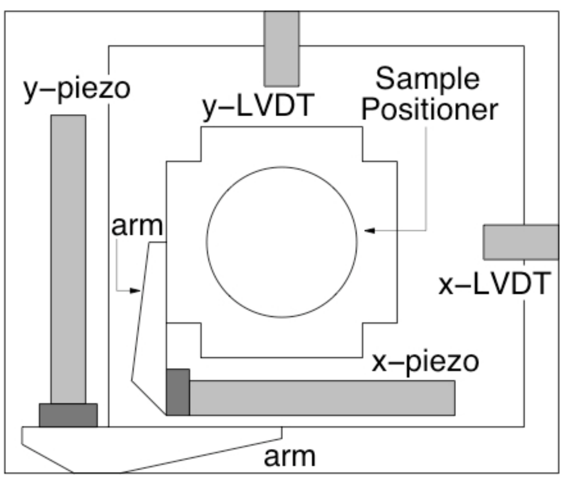


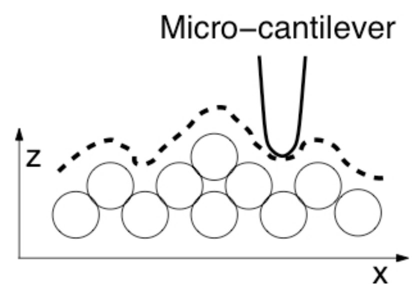


Atomic Force Microscope

Current Research: See Murti Salapaka's work at
http://nanodynamics.ece.umn.edu/research/highlights/highlightsAFM.htm

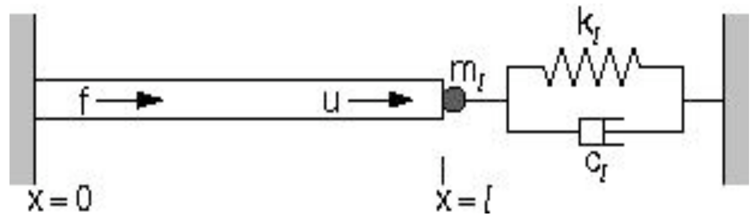


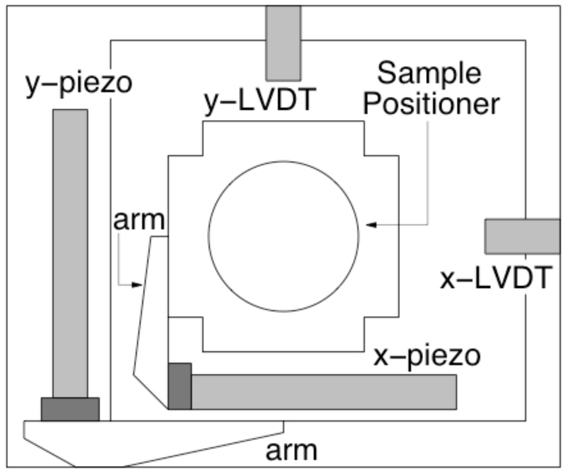




Modeling Process

Goal: Model Terfenol and AFM stage dynamics





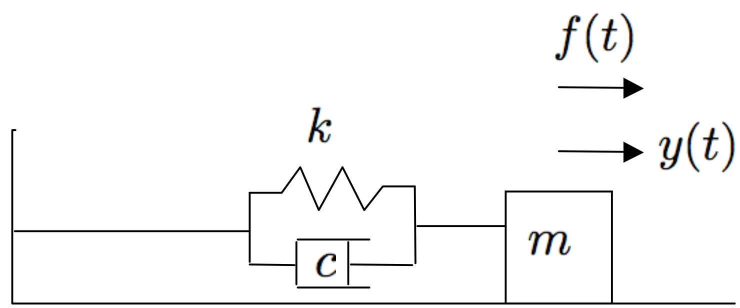
Strategy:

- Use physical understanding to make appropriate assumptions; e.g. uniform longitudinal forces permit use of lumped or spring model.
- Apply physical principles to develop model; e.g., Newtonian (force and moment balancing), Lagrangian (variational principles based on kinetic and potential energy), or Hamiltonian (total energy principles).
- Obtain analytic or numerical solution to model.
- Compare to experimental data (validate and predict).
- Update model to accommodate missing physics or inappropriate assumptions.

Derivation of Spring Model

Newtonian Principles: Balance forces using Newton's second law

- External Force: $f(t)$
- Spring Force: $F_s(t) = -ky(t)$
- Damping Force: $F_d(t) = -c\frac{dy}{dt}$



Newton's Second Law: $m\frac{d^2y}{dt^2} = F_s(t) + F_d(t) + f(t)$

Spring Model: $m\dfrac{d^2y}{dt^2} + c\dfrac{dy}{dt} + ky = f$

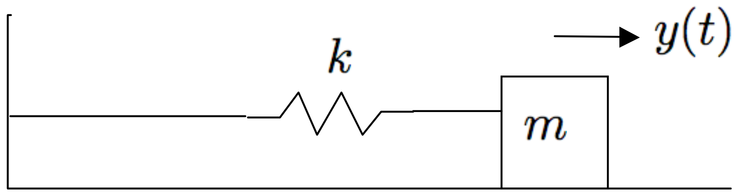
Initial Conditions: $y(0) = y_0 \, , \; \dfrac{dy}{dt}(0) = v_0$

Note: Details regarding classical mechanics can be found in Appendix C of the supplemental material.

Derivation of Spring Model

Lagrangian Principles: Take $c = f = 0$

- Kinetic Energy: $K(\dot{y}) = \frac{1}{2}m\dot{y}^2$
- Potential Energy: $$U(y) = \int_0^y kx\,dx = \frac{1}{2}ky^2$$



- Lagrangian: $$\mathcal{L}(y, \dot{y}, t) = K(\dot{y}) - U(y) = \frac{1}{2}m\dot{y}^2 - \frac{1}{2}ky^2$$
- Euler-Lagrange Equations: $$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} - \frac{\partial \mathcal{L}}{dy} = 0$$ $$\Rightarrow m\ddot{y} + ky = 0$$

Note: Details regarding calculus of variation and Lagrangian and Hamiltonian principles can be found in Appendix C of the supplemental material.

Analytic Solution of the Spring Model

Second-Order Model:

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = f$$

$$y(0) = y_0 \ , \ \frac{dy}{dt}(0) = v_0$$

Homogeneous Model: $f(t) = 0$

- Solve characteristic equation to obtain homogeneous solution $y_h(t)$
- Eigenvalue solutions of first-order system

Nonhomogeneous Model:

- Obtain particular solution $y_p(t)$ and general solution $y(t) = y_h(t) + y_p(t)$
  - Method of undetermined coefficients: e.g., $f(t) = \cos(\omega t)$
  - Variation of parameters: e.g., $f(t) = \ln t$
- Laplace transform: e.g., $f(t) = \delta(t - t_0)$

Analytic Solution of the Spring Model

First-Order System: Take $z_1 = y, z_2 = \dot{y}$

$$\Rightarrow \begin{bmatrix} \dot{z}_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f(t)/m \end{bmatrix}$$

$$\Rightarrow \dot{z}(t) = Az(t) + F(t)$$

Initial Condition: $z(0) = z_0$

Analytic Solution:

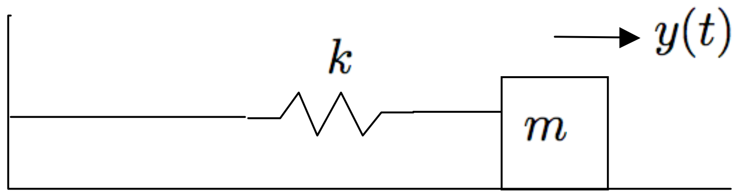
$$z(t) = e^{At} z_0 + \int_0^t e^{A(t-s)} F(s) ds$$

Importance:

- Analytic solution techniques
- Numerical approximation
- Control design

Analytic Solution of the Spring Model

Example: Take $c = f = 0$



$$y(t) = e^{rt} \Rightarrow mr^2 + k = 0$$

$$\Rightarrow r = \pm i\sqrt{k/m}$$

$$\Rightarrow y(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)\ ,\ \omega_0 = \sqrt{k/m}$$

- Eigenvalues and eigenvectors of A

$$\lambda_{1,2} = \pm i\sqrt{k/m} \qquad v_{1,2} = \begin{bmatrix} \pm 1 \\ i\sqrt{k/m} \end{bmatrix}$$

- Solution

$$z(t) = A\begin{bmatrix} \cos\omega_0 t \\ -\omega_0 \sin\omega_0 t \end{bmatrix} + B\begin{bmatrix} \sin\omega_0 t \\ \omega_0 \cos\omega_0 t \end{bmatrix}$$

Numerical Solution of the Spring Model

Consider First-Order System:

$$\Rightarrow \begin{bmatrix} \dot{z}_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ f(t)/m \end{bmatrix}$$

$$\Rightarrow \dot{z}(t) = Az(t) + F(t)$$

$$z(0) = z_0$$

Note: See notes for initial value problems (IVP)