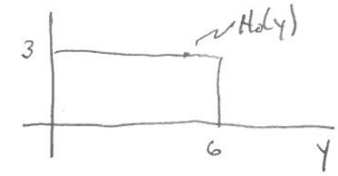
Exam 1

1a i) The flux is $q = v \cdot H = \frac{1}{2}\frac{m}{s} \cdot 3 \frac{\text{people}}{m} = \underline{\frac{3}{2}\frac{\text{people}}{s}}$. The model is

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial y} = 0, \quad 0<y<L$$
$$H(0,y) = H_0(y).$$



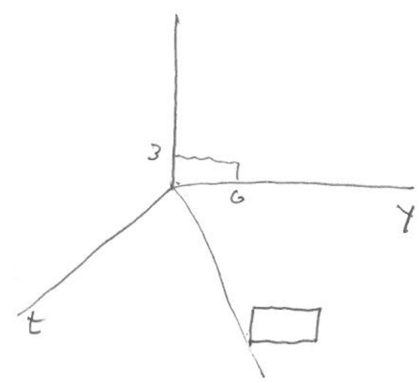
The solution is

$$H(t,y) = H_0(y - vt).$$

Note: The first patients reach the room at $t_1 = 30s = \frac{1}{2}$ min.

ii) The model here is

$$\frac{\partial H}{\partial t} - D\nabla^2 H = 0$$
$$H(0,x,y) = 0$$
$$\nabla H \cdot \hat{n}|_{\Gamma} = 0$$
$$\frac{\partial H}{\partial y}(t,x,y)\Big|_{(x,y)\in\Gamma} = b(t) = \begin{cases} \frac{3}{2}, & t_1 \le t \le t_2 = 42s \\ 0, & \text{else.} \end{cases}$$



Assumption: Approximate discrete behavior by continuous model

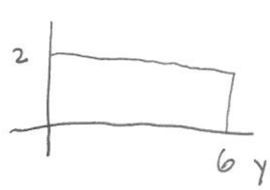
iii) They will spread throughout the room.

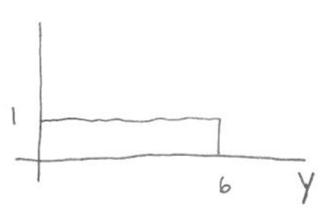
b i) Here

$$q_H = \frac{1}{2} \cdot 2 = 1 \frac{\text{people}}{s}$$
$$q_S = \frac{1}{2} \cdot 1 = \frac{1}{2} \frac{\text{people}}{s}$$

Model:

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial y} = 0 \qquad \frac{\partial S}{\partial t} + v\frac{\partial S}{\partial y} = 0$$
$$H(0,y) = H_0(y) \qquad S(0,y) = S_0(y)$$





ii) The model is

$$\frac{\partial H}{\partial t} - D\nabla^2 H - D\nabla^2 S = 0$$

$$\frac{dS}{dt} - D\nabla^2 S + D\nabla^2 H = 0$$

$$H(0,x,y) = 0$$

Similar initial and boundary conditions

$$\nabla H \cdot \hat{n}\big|_{\Gamma} = 0 \;, \quad \frac{\partial H}{\partial y}(t,x,y)\Big|_{(x,y)\in\Gamma} = b_H(t)$$

$$= \begin{cases} 1 & , t_1 \leq t \leq t_2 \\ 0 & , \text{else} \end{cases}$$

2) The time-dependent model is

$$c_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + S \qquad , \; q = k\frac{\partial T}{\partial x}$$

$$\Rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + s \qquad , \; \alpha = \frac{k}{c_p \rho} \;, \; s = \frac{S}{c_p \rho}$$

$$T(t,0) = T_0$$

$$k\frac{\partial T}{\partial x}(t,L) = 0$$

b) Here

$$\frac{d^2 T}{dx^2} = \frac{-s}{\alpha}$$

$$T(0) = T_0$$

$$\frac{\partial T}{\partial x}(L) = 0$$

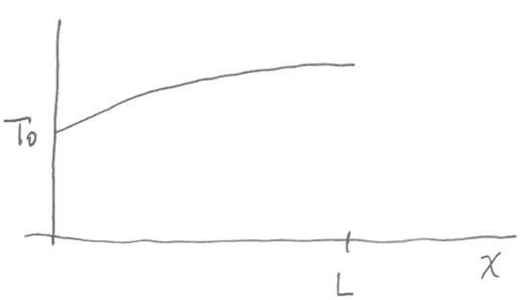
so the solution is

$$\frac{dT}{dx} = -\frac{s}{\alpha}x + C_1 \quad ; \; \frac{dT}{dx}(L) = \frac{-sL}{\alpha} + C_1 = 0 \Rightarrow C_1 = \frac{sL}{\alpha}$$

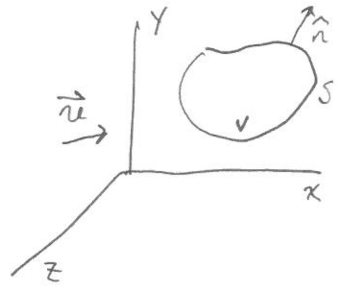
$$= \frac{s}{\alpha}x + \frac{sL}{\alpha}$$

and

$$T(x) = -\frac{s}{2\alpha}x^2 + \frac{sL}{\alpha}x + T_0$$



3) Here



$$\frac{d}{dt}\int_V \rho \, dV = -\int_S \hat{n}\cdot(\rho\vec{u})\,ds$$

$$\Rightarrow \int_V \left[\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u})\right]\partial V = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$