Math 573: Exam 1 — Fall 2017

(1) We are going to model a trip to the physician's office :(Consider an office having the shape depicted in Figure 1. Throughout this problem, we are going to assume that a person occupies 1 m^2 so the density and number of individuals coincide. Let H and S respectively denote the number (density) of healthy and sick individuals. At $t = 0$, 18 patients, lined in groups of three, begin walking down the hall Ω_1, of length $L = 21$ m, at a velocity of $v = 0.5$ m/s. You can assume that in the hall, there is no diffusion.

(a) We are first going to consider the case when all of the people are healthy and visiting the doctor for a checkup.

(i) Determine the flux q in the hall Ω_1 and develop a model quantifying the movement of people down the hall (be sure to specify and check the units of q). Determine and plot your solution. When do the first patients reach the room? Note that this will be a 1-D model and solution.

(ii) Within the room Ω_2, you can assume that movement is solely due to diffusion with constant D. The region where people enter the room is denoted by γ and the remaining walls are denoted by $\Gamma = \bigcap_{i=1}^{4} \Gamma_i$. Determine a model quantifying the density (movement) of people in the room. Be sure to specify appropriate boundary and initial conditions. What assumptions are you making when constructing your model?

(iii) Without solving the model, describe how people will be oriented after an hour.

(b) Now consider an initial group comprised of 12 healthy individuals and 6 sick individuals lined up with 1 sick individual in each column. Once the group enters the room, you can assume that healthy individuals diffuse away from sick ones while sick folks move toward healthy ones. Repeat the analysis in Part (a) and model the movement of the individuals.

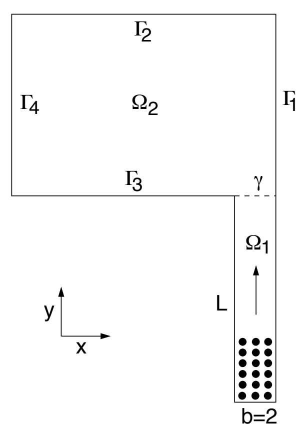

Figure 1: Orientation of the hall and room for Problem 1.

(2) Consider a rod of length L as depicted in Figure 2. The left end is maintained at a fixed temperature T_0 and the right end is perfectly insulated. The specific heat, density and thermal diffusivity, having units of J/(kg·K), kg/m^3, and m^2/s, are denoted by c_p, ρ and α. Furthermore, we assume that a *uniform* heat source S, with units of J/(m^3·s), is present along the entire length of the rod.

(a) Derive the time-dependent model quantifying the temperature distribution in the rod. Be sure to specify the flux and boundary conditions.

(b) The heat distribution in the rod will eventually reach a steady state and hence be independent of time. Simply your model from (a) to quantify this steady state behavior, and obtain and plot your solution. Does it appear feasible? Show that the heat flux through the left end is equal to the total heat supply. Explain why this is to be expected.

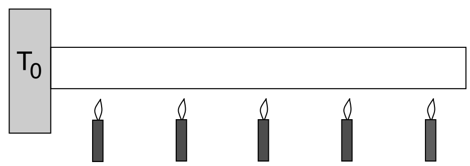

Figure 2: Rod having a uniform external heat source S with the left end attached to a heat reservoir and the right end insulated.

(3) Derive conservation of mass in 3-D and be sure to motivate each step.