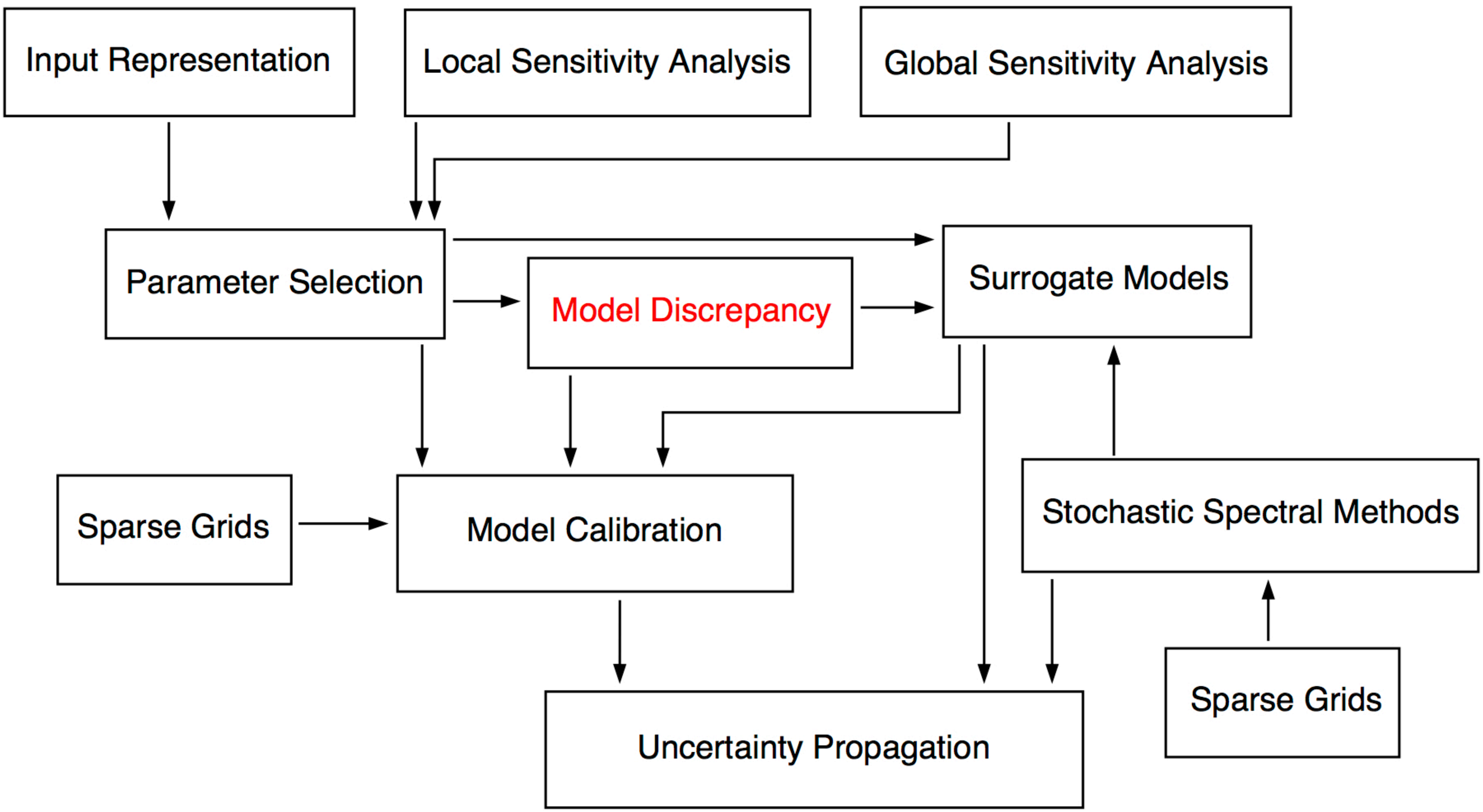
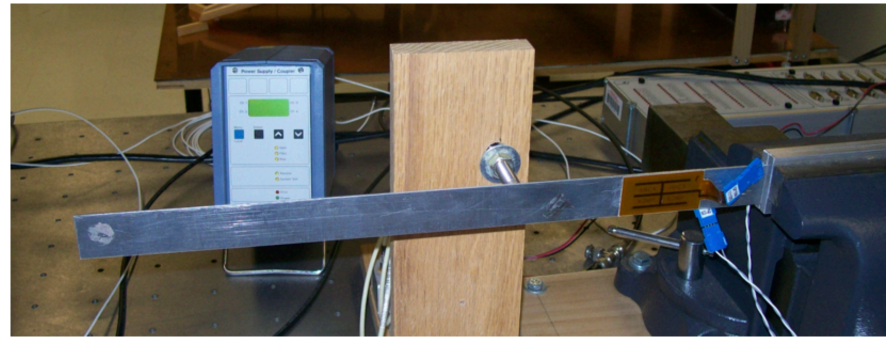
Steps in Uncertainty Quantification



6. Quantification of Model Discrepancy – Thin Beam

"Essentially all models are wrong, but some are useful" George E.P. Box

Example: Thin beam driven by PZT patches



Euler-Bernoulli Model: For all $\phi \in V$

$$\int_0^L \left[\rho(x)\frac{\partial^2 w}{\partial t^2} + \gamma \frac{\partial w}{dt}\right]\phi dx + \int_0^L \left[YI(x)\frac{\partial^2 w}{\partial x^2} + cI(x)\frac{\partial^3 w}{\partial x^2 \partial t}\right]\phi'' dx$$

$$= k_p V(t) \int_{x_1}^{x_2} \phi'' dx$$

with

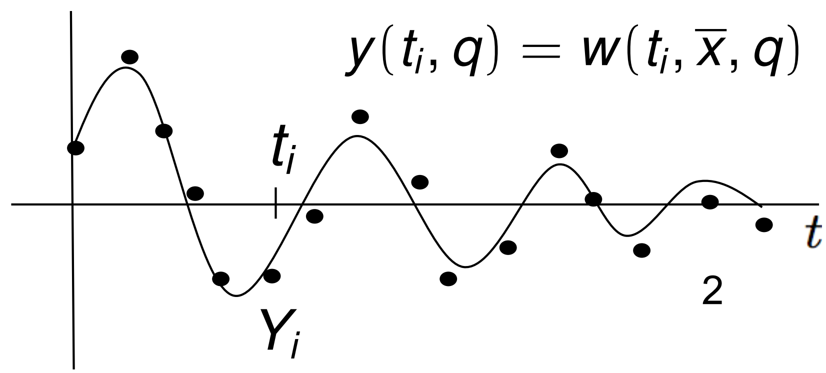
$$\rho(x) = \rho h b + \rho_p h_p b_p \chi_p(x) \;,\;\; YI(x) = YI + Y_p I_p \chi_p(x)$$

$$cI(x) = cI + c_p I_p \chi_p(x)$$

Note: 7 parameters, 32 states

Statistical Model:

$$Y_i = y(t_i, q) + \varepsilon_i$$

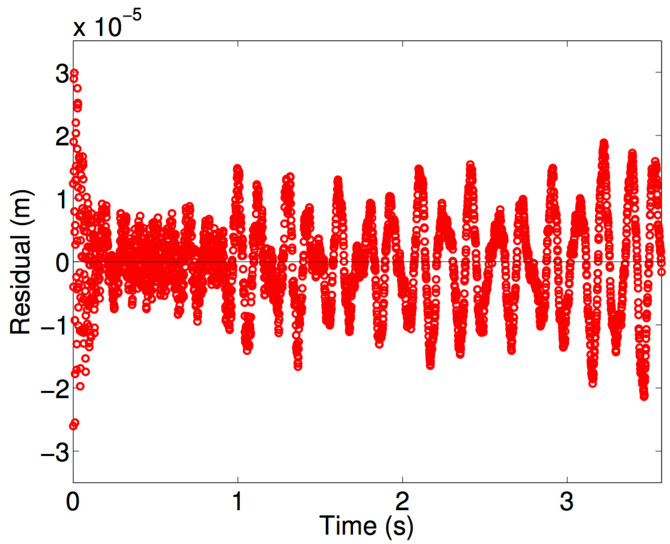


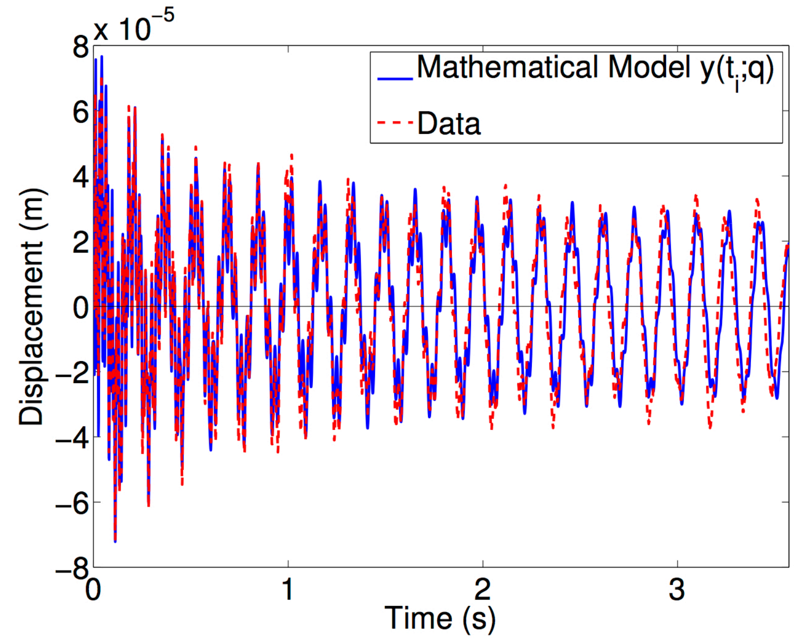
Quantification of Model Discrepancy – Thin Beam

Example: Good model fit

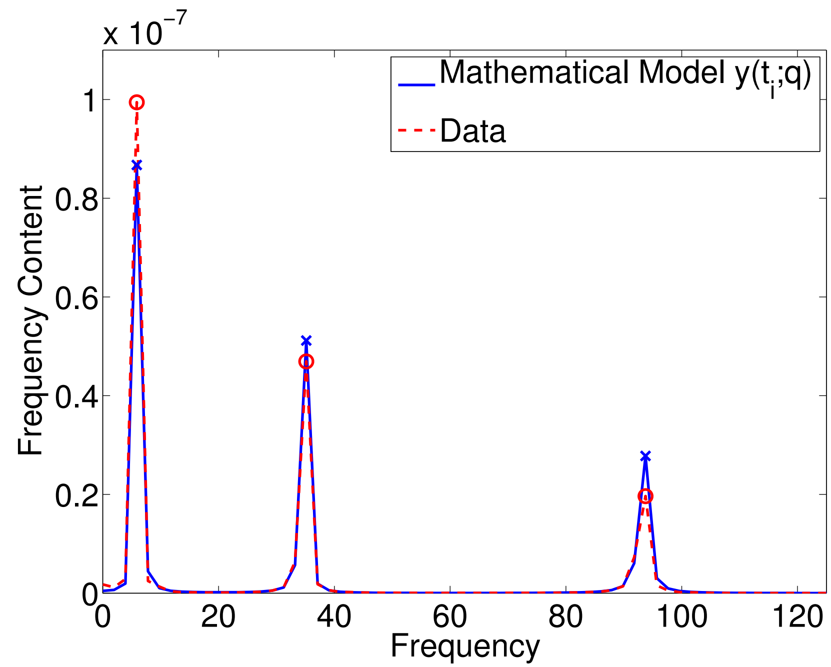
$$Y_i = y(t_i, q) + \varepsilon_i$$

Note: Observation errors not iid





Model Fit to Data



Reference: Additive observation errors

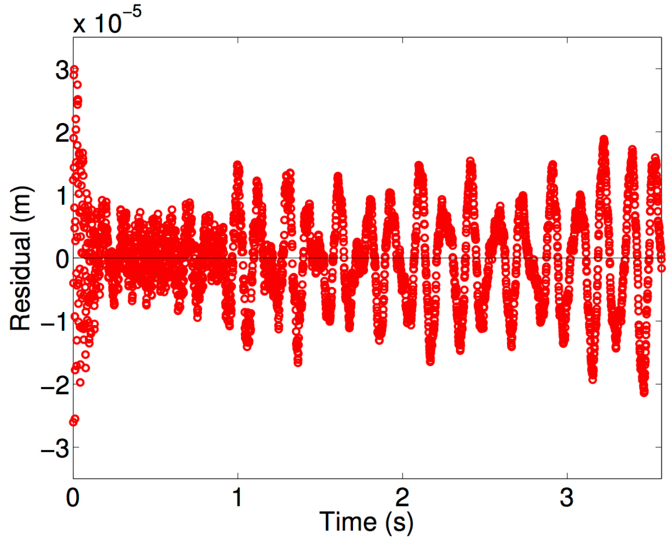
$$Y_i = y(t_i, q) + \delta(t_i, \widetilde{q}) + \varepsilon_i$$

- M.C. Kennedy and A. O'Hagan, *Journal of the Royal Statistical Society, Series B*, 2001.

Quantification of Model Discrepancy – Thin Beam

Example: Good model fit

Problem: Observation errors not iid



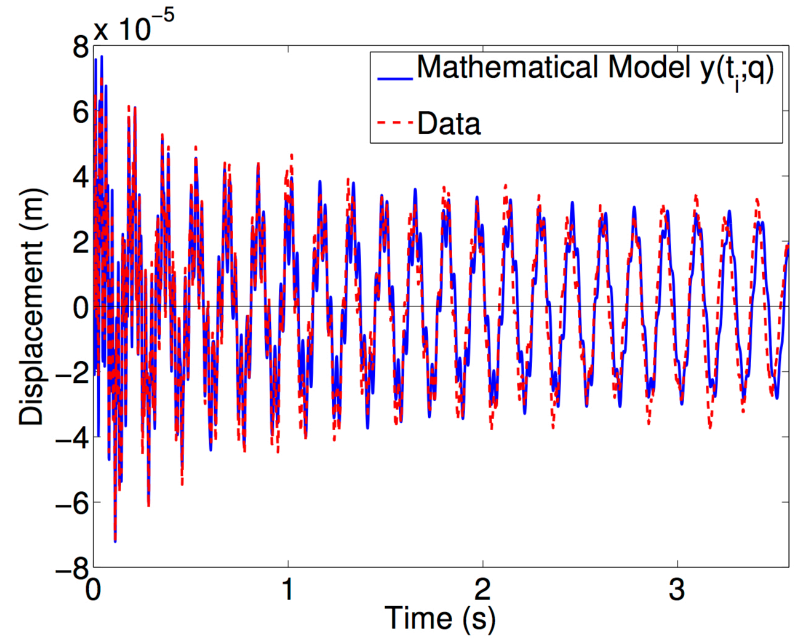
Result: Prediction intervals wrong

Approaches:

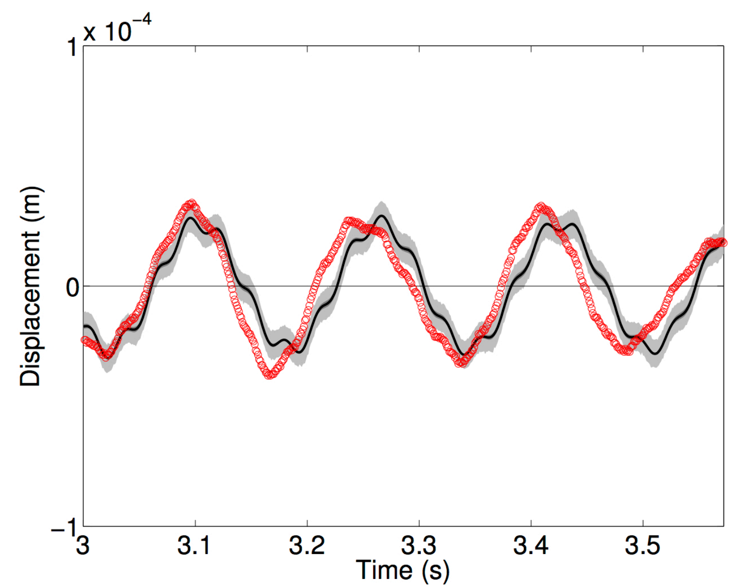
• GP Model: Inaccurate for extrapolation

• Control-based approaches: difficult to extrapolate.

• Problem: correct physics or biology required for extrapolation!



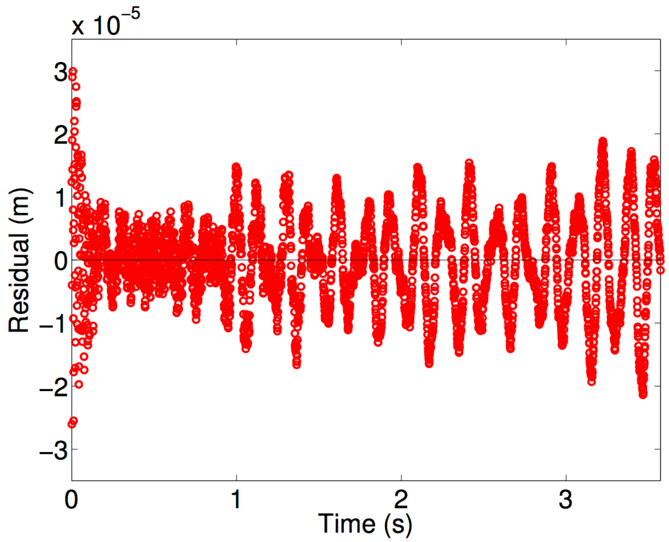
Model Fit to Data

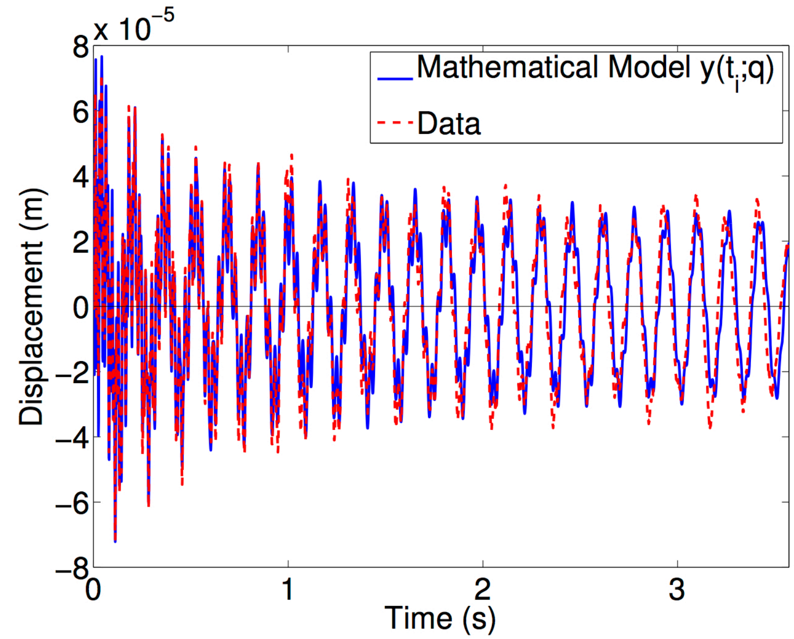


Prediction Intervals and Data

Quantification of Model Discrepancy – Thin Beam

Problem: Measurement errors not iid





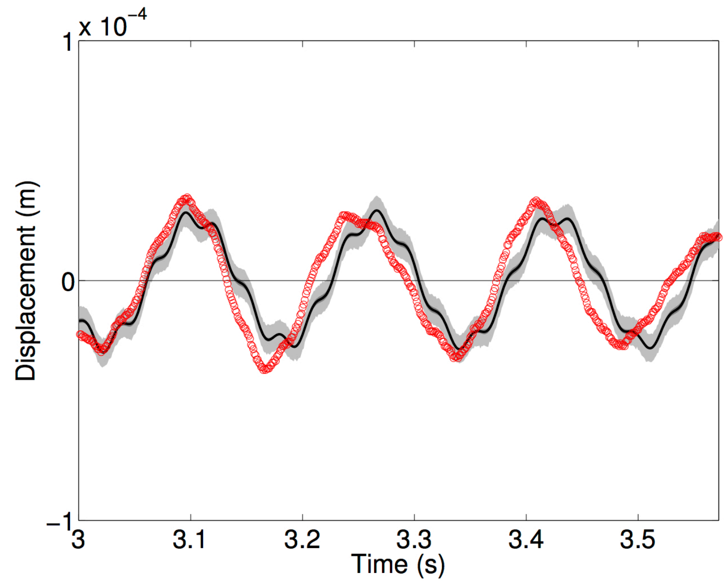
Model Fit to Data

Result: Prediction intervals wrong

One Approach:

• Determine components of model you trust (e.g., conservation laws) and don't trust (e.g., closure relations). Embed uncertainty into latter.

• T. Oliver, G. Terejanu, C.S. Simmons, R.D. Moser, *Comput Meth Appl Mech Eng*, 2015.



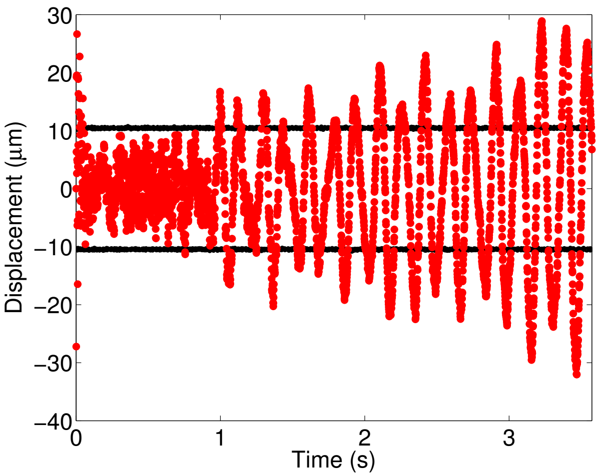
Prediction Intervals and Data

2018-19 SAMSI Program: Model Uncertainty: Mathematical and Statistical (MUMS)

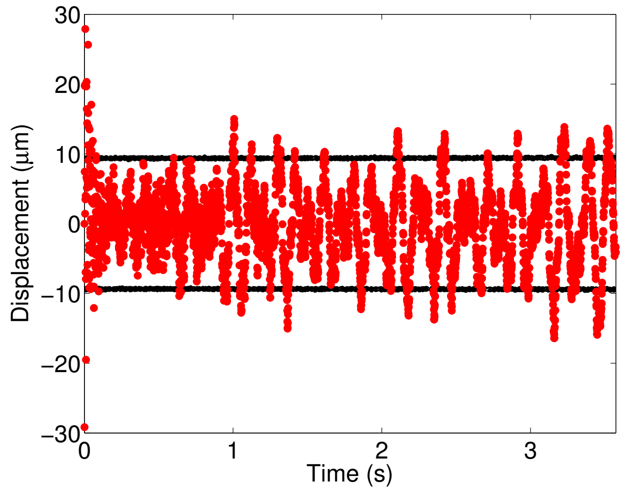
Quantification of Model Discrepancy – Thin Beam

Our Solution: “Optimize” calibration interval

- Use damping/frequency domain results to guide.



Calibrate on [0,1]



Calibrate on [0.25,1.25]

Note: We have substantially extended calibration regime.