Math 540: Project 4

Due Thursday, March 28

1. Consider the SIR model

$$\begin{aligned}
\frac{dS}{dt} &= \delta N - \delta S - \gamma IS \quad , \quad S(0) = 900, \\
\frac{dI}{dt} &= \gamma IS - (r+\delta)I \quad , \quad I(0) = 100, \\
\frac{dR}{dt} &= rI - \delta R \quad , \quad R(0) = 0
\end{aligned}$$

from Project 3, where γ, r and δ are each in the interval $[0, 1]$.

(a) Consider the file `SIR.txt`, which contains times t_j in the first column and corresponding values $I(t_j)$ in the second, and the parameters $q = [\gamma, r, \delta]$. Employ DRAM to compute and plot chains, marginal densities, and pairwise plots for the parameters. You should use `s2chain` to additionally estimate the observations variance σ^2. You can use the covariance matrix V, which you estimated in Project 3, as input. How do the mean parameter values compare to the OLS estimates that you computed using `fminsearch.m`? How does the final adapted covariance matrix compare to your initial estimate V? Plot the marginal densities and sampling distributions in the same figures and discuss your results. Finally, how does the variance estimate σ^2 computed by DRAM compare to your OLS estimate?

(b) Using the DRAM commands `mcmcpred` and `mcmcpredplot`, construct 95% credible and prediction intervals for each of the states. Do your results appear to be reasonable?

(c) Now consider the model

$$\begin{aligned}
\frac{dS}{dt} &= \delta N - \delta S - \gamma kIS \quad , \quad S(0) = 900, \\
\frac{dI}{dt} &= \gamma kIS - (r+\delta)I \quad , \quad I(0) = 100, \\
\frac{dR}{dt} &= rI - \delta R \quad , \quad R(0) = 0
\end{aligned}$$

non-identifiable parameter set $q = [\gamma, r, \delta, k]$. Run DRAM and plot the pairwise distributions. Are your chains converging? Can you use the Bayesian analysis to establish which parameters are not mutually identifiable?

2. Consider the Helmholtz energy

$$\psi(P, q) = \alpha_1 P^2 + \alpha_{11} P^4 + \alpha_{111} P^6$$

from Project 3, where P is the polarization on the interval $[0, 0.8]$ and $q = [\alpha_1, \alpha_{11}, \alpha_{111}]$ are parameters.

(a) Using the data in the file `Helmholtz.txt`, which contains polarization values P_j in the first column and energies $\psi(P_j)$ in the second, employ DRAM to compute chains, marginal densities, and pairwise plots for the parameters. Please report your parameter estimates and observation variance σ^2. Discuss the correlation of the parameters and why the global sensitivity analysis techniques in Project 2 are not applicable.

(b) Using the DRAM commands `mcmcpred` and `mcmcpredplot`, construct 95% credible and prediction intervals for the energy and discuss your results.

(c) Now repeat (a) using the Random Walk Metropolis Algorithm 8.5. How do these results compare to DRAM?