Math 540: Project 5

Due Thursday, April 19

1. Consider the Helmholtz energy

$$\psi(P, q) = \alpha_1 P^2 + \alpha_{11} P^4 + \alpha_{111} P^6 \quad , \quad q = [\alpha_1, \alpha_{11}, \alpha_{111}],$$

evaluated at the polarization value $P = 0.8$, as your response. Using the parameter means and covariance matrix that you inferred in Problem 2 of Project 4, analytically compute the mean and variance of your response. How do the mean and 2σ compare with the credible interval at $P = 0.8$, which you computed in Project 4?

2. Exercise 10.5. You can use the code posted on the book website to run the discrete projection and Monte Carlo methods. Fir stochastic collocation, you should compute the mean and variance in two ways. First, you can construct and solve the Vandemonde system (10.30) with $K = 9$ to obtain the coefficients u_k, which you can employ in (10.9) and (10.10) to compute the mean and variance. Be sure to compute the conditioning of the matrix to establish the validity of solving the system. Secondly, compute the two quantities based on a Lagrange expansion for the interpolant.

3. Repeat Example 10.13 with the assumption that $m \sim \mathcal{U}(\bar{m} - 0.1, \bar{m} + 0.1), c \sim \mathcal{U}(\bar{c} - 0.1, \bar{c} + 0.1)$ and $k \sim \mathcal{U}(\bar{k} - 0.1, \bar{k} + 0.1)$, where $\bar{m} = 2.7, \bar{c} = 0.24$ and $\bar{k} = 8.5$. You can accomplish this by modifying the posted code. Specifically, you will change the random variables from Gaussian to uniform and replace the Gauss-Hermite points and weights by Gauss-Legendre points and weights on the interval $[-1, 1]$. You will also need to modify the basis functions from Hermite to Legendre.